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3D-Computaional Mesh Generation around a Propeller by Elliptic Differential Equation System

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Abstract

In this paper, an analytic method is applied for the generation of 3D mesh system in order for Navier-Stokes equations around ATP (Propfan). One of the advantages of this method is that mesh lines have strong differentiability. The differential equation used is Poisson type and the right hand side is called control function, and this is able to control the degree of mesh line clustering. Here, the form of control function was contrived to cluster near the solid surfaces. By this method, several mesh lines are laid in the boundary layer above the blade surfaces.

1. Introduction

Due to very rapid progress of large scale super-computers, the nonlinear partial differential equations in fluid dynamics, e. g. Euler equation and Navier-Stokes equation, have been possible to be solved numerically. Almost all methods of solutions adopt finite difference calculations in which the flow field is divided into many polyhedrons for three-dimensional case and the differential equations are transformed into difference equations. Accordingly, method of division of the flow field is very important, since generated computational mesh largely affects the stability and convergence of the solutions, and, moreover, the physical quantities solved change their values whether the mesh is good or bad.

We can find a lot of papers[1~7] about the mesh generation techniques cited in the references. These techniques are divided into two groups as follows:

(i) Algebraic generation method.

(ii) Analytic generation method.

By the method (i), the every grid points and coordinate curves can be determined arbitrarily, while the curves are determined by the solutions of elliptic or hyperbolic differential equations in method(ii). Therefore, the location of coordinate curves can be done easily, and, moreover, the normality condition upon the boundaries and the spacing control of mesh points in the specified region are easy. Against the method(i), the analytic method is not so easy to control the mesh lines, however, the strong differentiability can be kept along the mesh lines.

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In this paper, 3D computational meshes around ATP are generated. The Poisson equation (elliptic equation) is used for this purpose. The right hand side of this equation, called control function, controls the mesh line locations.

2. Basic Equations

Let us try to map the physical space between two blades of an ATP into the computational space as shown in Fig.1. The physical space, xyz-space, is surrounded by two blades, nacelle, outer boundary, inflow and outflow boundaries, and the computational space, $\xi\eta\zeta$-space, is a rectangular pararellpiped. This is divided into $N_\xi \times N_\eta \times N_\zeta$ small pararellpipedes with each length as $\Delta\xi$, and $\Delta\zeta$. The mapping of this computational space into boundary fitted mesh lines in the physical space is performed.

The elliptic differential equation is applied for the governing equation, and the mapping is completed by solving a boundary value problem.

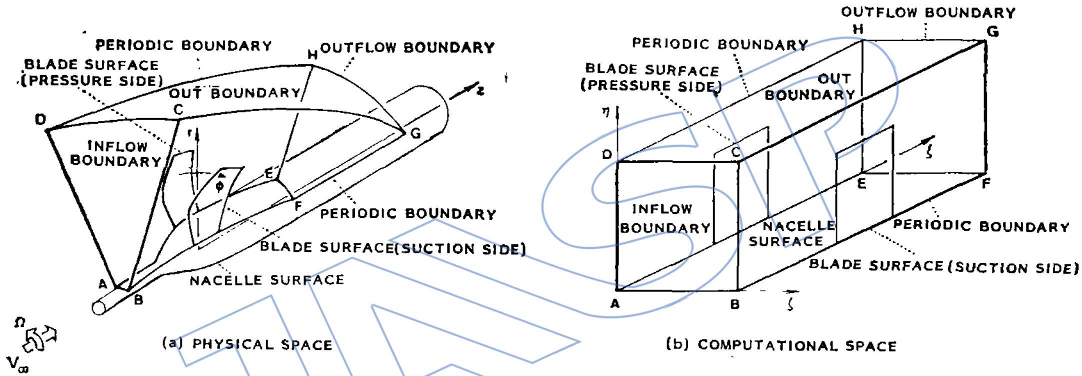

Fig.1 Space around ATP.

$$\nabla^2 \xi^i = 0, \qquad \nabla^2 = \sum_{k=1}^{3} \frac{\partial^2}{\partial x_k} \tag{1}$$

where $\xi^1 = \xi$, $\xi^2 = \eta$ and $\xi^3 = \zeta$

Eq.(1) is Laplace equation, however, the following Poisson equation is more useful for this purpose.

$$\nabla^2 \xi^i = P^i, \qquad i = 1, 2, 3 \tag{2}$$

where P^i is called the control function, and the coordinate curves are able to be controlled by this value. For negative P^i, a constant ξ^i surface in the physical space is attracted to the smaller value of x_i, where $x_1 = x$, $x_2 = y$ and $x_3 = z$ and vice versa. In a word, the mesh lines can be dense or coarse due to the negative or positive values of the right hand side of Eq.(2).

Changing the dependent and independent variables with each other, Eq.(2) is rewritten as follows:

$$\sum_{i=1}^{3}\left(\sum_{j=1}^{3} g_{ij}\frac{\partial^2 x_K}{\partial\xi^i\partial\xi^j}+p^i\frac{\partial x_K}{\partial\xi^i}\right)=0 \tag{3}$$

where $g^{ii}=(g_{km}g_{ln}-g_{kn}g_{lm})/g$

$$g_{ij}=\underset{\sim}{r}{}_{\xi}^{i}\cdot\underset{\sim}{r}{}_{\xi}^{j},\quad \underset{\sim}{r}=[xyz]^T,\qquad \underset{\sim}{r}{}_{\xi}^{i}=\left[\frac{\partial x}{\partial\xi^i}\ \frac{\partial y}{\partial\xi^i}\ \frac{\partial z}{\partial\xi^i}\right]^T$$

$$g=[\underset{\sim}{r}_{\xi 1}\cdot(\underset{\sim}{r}_{\xi 2}\times\underset{\sim}{r}_{\xi 3})]^2$$

Moreover, put

$$p^i=g^{ii}p^i \tag{4}$$

Eq.(3)is written as follows,

$$\sum_{i=1}^{3}\sum_{j=1}^{3} g^{ij}\left(\frac{\partial^2 x_k}{\partial\xi^i\partial\xi^j}+\delta_{ij}p^i\frac{\partial x_k}{\partial\xi^i}\right)=0 \tag{5}$$

3. Discritization and Method of Solution

As described in the previous section, the desired mesh is obtained by solving a boundary value problem of Eq.(5). It is needless to say that the method of solution for Eq.(5) is numerical, then, space differentiation is expressed by finite difference. The central difference with second accuracy is used herein, i.e.

$$\frac{\partial x}{\partial\xi}=\frac{x(i+1,j,k)-x(i-1,j,k)}{2\Delta\xi}$$

$$\frac{\partial^2 x}{\partial^2\xi}=\frac{x(i+1,j,k)-2x(i,j,k)+x(i-1,j,k)}{\Delta\xi^2}$$

$$\frac{\partial^2 x}{\partial\xi\partial\eta}=\frac{x(i+1,j+1,k)-x(i-1,j+1,k)-x(i+1,j-1,k)+x(i-1,j-1,k)}{4\Delta\xi\Delta\eta}$$

etc. (6)

Substituting these expressions into Eq.(5), x(i,j,k) is solved as follows,

$$x(i,j,k)=w(d_1+d_2+d_3+d_4+d_5+d_6+g^{11}x_\xi P+g^{22}x_\eta Q+g^{33}x_\zeta R/2C_M \tag{7}$$

where

$$\left.\begin{aligned}
d_1&=\frac{g^{11}}{\Delta\xi^2}\{x(i+1,j,k)-x(i-1,j,k)\}\\
d_2&=\frac{g^{22}}{\Delta\eta^2}\{x(i,j+1,k)+x(i,j-1,k)\}\\
d_3&=\frac{g^{33}}{\Delta\zeta^2}\{x(i,j,k+1)+x(i,j,k-1)\}\\
d_4&=d_{12}(g^{12}+g^{21})\\
d_5&=d_{13}(g^{13}+g^{31})\\
d_6&=d_{23}(g^{23}+g^{32})
\end{aligned}\right\} \tag{7a}$$

$$\left.\begin{aligned}
d_{12} &= \frac{\partial^2 x}{\partial\xi\partial\eta} = \frac{1}{4\Delta\xi\Delta\eta}\{x(i+1,j+1,k) - x(i+1,j-1,k) \\
&\quad - x(i-1,j+1,k) + x(i-1,j-1,k)\} \\
d_{13} &= \frac{\partial^2 x}{\partial\xi\partial\zeta} = \frac{1}{4\Delta\xi\Delta\zeta}\{x(i+1,j,k+1) - x(i+1,j,k-1) \\
&\quad - x(i-1,j,k+1) + x(i-1,j,k-1)\} \\
d_{23} &= \frac{\partial^2 x}{\partial\eta\partial\zeta} = \frac{1}{4\Delta\eta\Delta\zeta}\{x(i,j+1,k+1) - x(i,j+1,k-1) \\
&\quad - x(i,j-1,k+1) + x(i,j-1k-1)\} \\
C_w &= \frac{g^{11}}{\Delta\xi^2} + \frac{g^{22}}{\Delta\eta^2} + \frac{g^{33}}{\Delta\zeta^2}
\end{aligned}\right\} \tag{7b}$$

In order to obtain the converged x(i, j, k), we used the iteration method with relaxation, i.e. $x^*(i,j,k)$ is calculated using n-th value x (i, j, k) by Eq.(7),and Δx is obtained by

$$\Delta x = x^*(i,j,k) - x(i,j,k) \tag{8}$$

Then, (n+1)-th value of x is obtained as follows:

$$x^{n+1}(i,j,k) = x^n(i,j,k) + \omega\Delta x \tag{9}$$

where ω is relaxation coefficient which makes convergence speed faster and solution be stable. The above process should be continued until Δx becomes smaller than the preassigned value.

4. Control Function

As described above, mesh lines can be controlled by the control function P^i (or P, Q, R). The purpose of this paper is to generate the mesh system which is applied for Navier-Stokes computational code around ATP. Therefore, the control function is used for the mesh line clustering near solid surface where the boundary layer is developing. Since the solid surfaces locate along ζ-constant plane, we can put

$$P = Q = 0 \tag{10}$$

Then, R is given by

$$R = -fa[\exp\{-b(\zeta-\zeta_1)\} - \exp\{-b(\zeta_N-\zeta)\}] \tag{11}$$

where

$$\left.\begin{aligned}
a &= -\frac{(\zeta-\zeta_1)(\zeta-\zeta_N)}{(\zeta_N-\zeta_1)^2}a' \\
b &= \frac{1}{\zeta_N-\zeta_1}b' \\
f &= f_1 \cdot f_2
\end{aligned}\right\} \tag{12}$$

In the above equation, f controls the mesh line location along ξ and η directions, by second order curves, i. e. f_1 and f_2 determine the density along ξ and η directions respectively shown in Fig.2. Moreover, a' and b' are determined by trial and error.

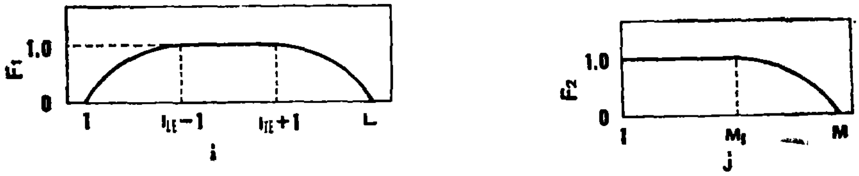

Fig. 2 Control functions

In order to determine the degree of clustering of the mesh lines, we must estimate the thickness of boundary layer. Since we only put the thin-layer assumption upon the blade surfaces for the solution of Navier-Stokes equation, several mesh lines may be laid in the boundary layer. Since the thickness of turbulent boundary layer is proportional to $Re^{-1/2}$, $\delta \cong 10^{-3}$ in the case of $Re \cong 10^{6}$. Accordingly, 10^{-4} is enough for the duration of two neighboring mesh lines.

5. Mesh System for Rotating Blades

Applying the above mentioned method to the space between two neighboring blades, we try to generate the computational mesh system around a propeller. By this mesh system, we aim to solve Navier-Stokes equations where thin boundary layer is assumed on the blade surfaces, while the flow slips on the nacelle surface, i.e. the viscous terms are neglected, then, the governing equations are Euler ones on the nacelle surface.

The numbers of mesh lines are 45, 18 and 21 along x, y and z-directions respectively, and we put a′ and b′ in Eq. (12) as

$$a' = 1.5 \times 10^{3}$$

$$b' = 1.25 \times 10$$

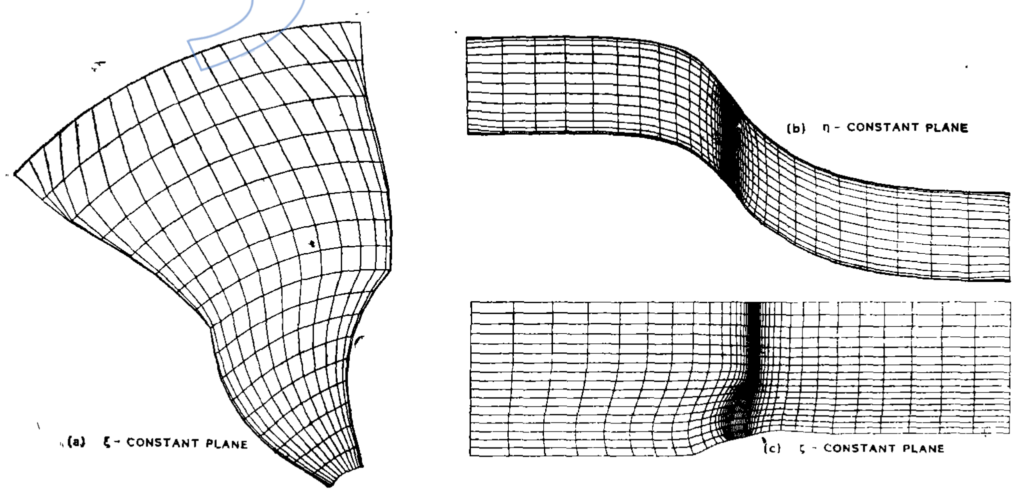

Fig.3 Mesh system around SR-3 ATP.

The initial mesh system were generated appropriately by algebraic method. Using this system, we started the iteration. An example of mesh after 20 itera ions are illustrated in Fig.3. Fig.4 shows mesh lines on the solid surfaces. The clustering of mesh lines near the blade surfaces is depicted in these figures.

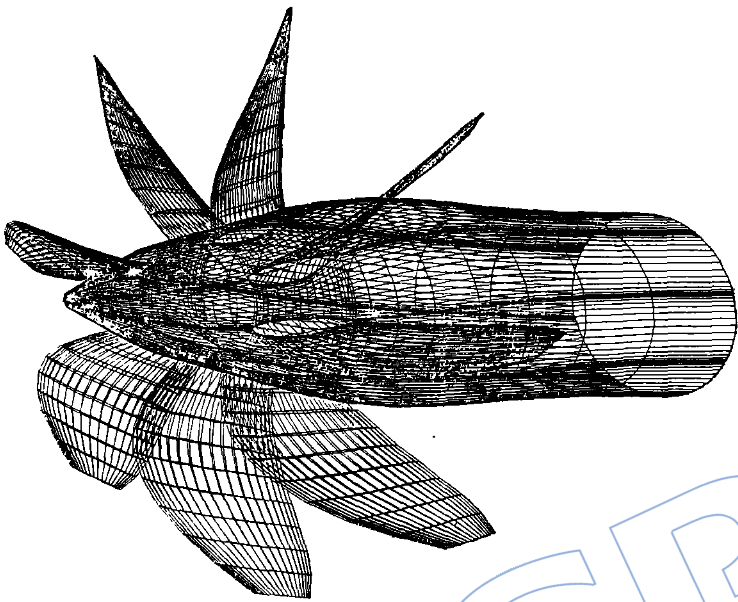

Fig. 4 Meshes on the surfaces of SR-3 ATP.

6. Conclusion

In order to solve Navier-Stokes equations numerically around ATP, it is necessary to generate computational mesh system suitable for the calculations. In this paper, an analytic method in which the elliptic differential equation is solved is applied for the generation of 3D mesh system. One of the advantages of this method is that mesh lines have strong differentiability. The differential equation used is Poisson type and the right hand side is called control function, and this is able to control the degree of mesh line clustering. Here, the form of control function was contrived to cluster near the solid surfaces. By this method, several mesh lines are laid in the boundary layer above the blade surfaces.

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